STATISTICAL OPTIMIZATION AND SENSITIVITY ANALYSIS ON MECHANICAL PROPERTY IN HYDRO FRACTURING PROCESS

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Abstract

In the hydrofraturing process, the parameters such as Pressure in N/mm^2, Temperature in ºC, Injection hole diameter in mm play a major role in determining the fracture length during the hydrofraturing process. A central composite rotatable design with three factors and three levels was chosen to minimize the number of experimental conditions. An empirical relationship was established to predict the fracture length(mm) of the hydrofraturing process by incorporating independently controllable hydrofraturing process parameters. Response surface methodology (RSM) was applied to optimize the process parameters to attain maximum fracture length (mm). Sensitivity analysis was also carried out to understand the impact of each process parameters on Fracture length.

Keywords: Hydro fracturing, fracture length, optimization, response surface methodology, sensitivity analysis

Introduction

The analysis of the hydro-mechanical behavior of rock masses remains an important topic in rock mechanics, due to it being a critical phenomenon in ongoing challenging issues such as tunneling under high groundwater pressures, extraction of hydrocarbons from deep, pressurized petroleum reservoirs, and underground nuclear waste disposal. Despite continuing and extensive efforts, such analysis continues to be difficult. Hydro-mechanical response in a rock mass is identified as the interaction between the solid phase of the rock materials and any interstitial fluid (Rutqvist.J 2003). This technique involves pumping a fluid under pressure into a borehole. This pressurized fluid introduced into the borehole produces stress concentration in the surrounding rock causing the development of fractures due to micro mechanical effects (B.Guruprasad 2012). Because of the heterogeneity of the material properties, rock structure and in situ stress state, the hydraulic fracturing process is highly complex (Germanovich.L.N 1997). A common difficulty in the hydraulic fracturing process in the real time is in observation and measurement of the fractures that develop beneath of the earth. Generally, the induced fracture geometry is measured by cutting the sample after the test (Murdoch, L 1993) (de Pater. C 1994) (Abass, H 1996) or by using an acoustic monitoring system (de Pater, C 1994) (Groenenboom, J 2000).

This method gives valuable results but limitations are there. The final results are observed by cutting the samples after the test. The resolution of the acoustic method is currently insufficient to capture details of the fracture propagation process. As a result, laboratory experiments on hydraulic fracturing in transparent materials have also been performed. These studies allowed the visualization in real time of the developing geometry of the fracture (Rummel F 1987) (Bunger A 2004) and the direction of fracture propagation (Hubbert M. K 1957) (Takada A 1990) (Bakala M 1997). Commonly used transparent geometrical analogues for fracturing are poly methyl methacrylate (PMMA, acrylic) (Rummel, F 1987) (Cooke M L 1996) . Since, the Fracture behavior is hard to predict because the relationship between stress and permeability is complex and highly dependent on pressure, temperature and Injection hole diameter. The resulting fractures can be used to analysis the basis of hydraulic fracture propagation in real time field applications, the developed empirical relationship can be effectively used to predict the Fracture length in millimeters of Hydro fracturing process.

In this Research paper, It is well known that the input of hydrofraturing process parameters play a major role in determining the fracture length. As the process facts have not been disclosed so far, the selection of input parameters to find the fracture length (mm) is very difficult. A common difficulty in the hydraulic fracturing process in the real time is in observation and measurement of the fractures that develop beneath of the earth. Hence, the problem of getting optimized hydrofraturing process parameters to attain maximum fracture length is attempted in this investigation. The Sensitivity analysis was also carried out to understand the impact of each process parameters on Fracture length during the hydrofraturing technique.

Experimental work

Fabricating the Experimental set up

The experimental set up [Fig.1] consists of a container for storing the fluid, a commercially available feed pump to feed pressurized fluid to the inner casing pipe provided in the PMMA test sample. The 20 nos. of PMMA test samples were prepared for the test. The PMMA test sample has a length of 300mm and outside diameter of 150mm. The inner casing pipe made up of stain less steel and inner diameter was 6 to 10 mm. The applied pressure can be varied manually by adjusting the two control valves provided in the experimental setup in the range of 4 to 8 N/mm^2. Before starting the experiment, the required pressure applied in to casing pipe is to be ensured by adjusting the flow control valves. A separate by pass line is provided in the experimental setup for achieving the required pressure for the same. A 555 timer IC is provided for feed pump to control the pressurized fluid rate with respect to the time, say 5 sec to 15 mins. The PMMA test sample is placed over the heater for heating purpose in the range of 40 to 60ºC. The heater control unit is made up of Nichrome heater having a capacity of 400W. The Dimmersat is 0-2A, Single phase, open type and it is provided for varying the input to the heater and measurement of input is carried out by a voltmeter, ammeter. The Voltmeter – Digital range is 0 to 200V AC, The Ammeter digital range is 0 to 2A AC, The temperature indicator is digital 0 to 199.9ºC. The electrical supply for the experimental setup is AC single phase, 230V earthed stabilized current. By varying the Dimmerstat, adjust the heat input at desired value for desired temperature on the PMMA sample. The commercially available thermocouples are embedded to the PMMA test sample for temperature measurement through a temperature gauge. The experimental table and Stand made up of MS square hollow pipe and angle. The pressure applied in the range of 4 to 8 N/mm^2 to the casing pipe, the temperature range for the study is 40ºC to 60ºC and the casing pipe diameter is 6mm to 10mm (B.Guruprasad 2012).

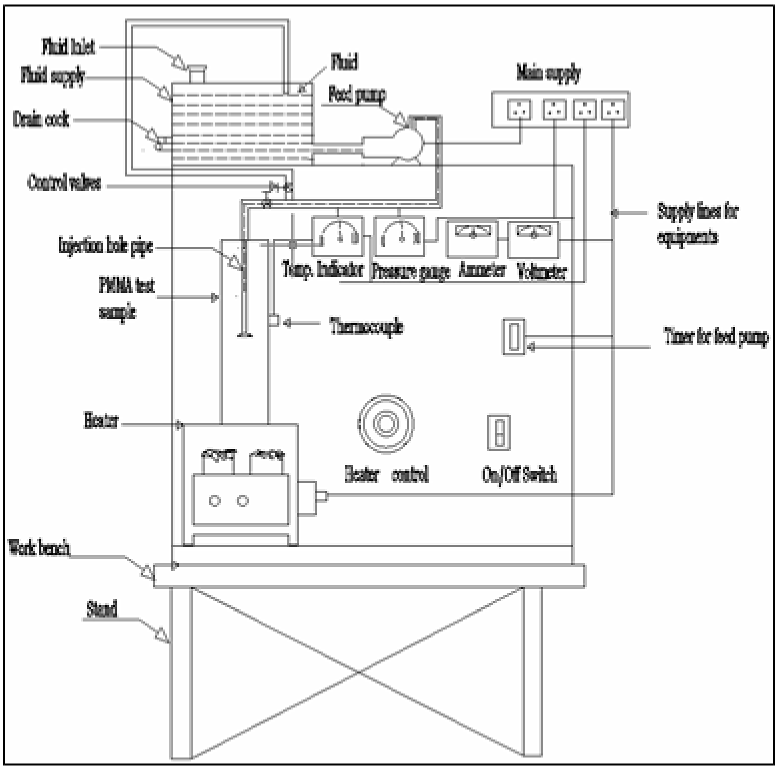

Fig.1 Experimental set up for Hydrofraturing process

From the literature, the predominant factors that have a greater influence on the Fracture rate of Hydro fracturing process had been identified. They were: (i) Pressure applied in N/mm^2 (ii) Temperature in ºC (iii) Injection hole diameter in mm. Large numbers of trial experiments were conducted to identify the feasible testing conditions for obtaining the Fracture length of Hydro fracturing process. The following inferences were obtained:

1. Based on the field trials the pressure applied is limited to 4 to 8 N/mm^2.
2. From the literature survey, the temperature and the injection hole diameter is limited to the range of 40 to 60 ºC and 6 to 10 mm respectively.
3. Further the Maximum with stand temperature of the PMMA samples is to be less than 100ºC, hence the temperature range is fixed to 40 to 60 ºC only (B.Guruprasad 2012)

Developing the experimental design matrix

Owing to a wide range of factors, the use of three factors and a central composite rotatable design matrix were chosen to minimize the number of experiments. A design matrix consisting of 20 sets of coded conditions (comprising a full replication three factorial of 8 points, six corner points and six center points) was chosen in this investigation. Table 1 represents the range of factors considered, and Table 2 shows the 20 sets of coded and actual values used to conduct the experiments. For the convenience of recording and processing experimental data, the upper and lower levels of the factors were coded here as +1.682 and -1.682 respectively. The coded values of any intermediate value could be calculated using the following relationship.

$$Xi = 1{:}682[(2X - (Xmax - Xmin)] / [Xmax - Xmin] \quad (1)$$

Where Xi is the required coded value of a variable X and X is any value of the variable from Xmin to Xmax, Xmin is the lower level of the variable, Xmax is the upper level of the variable.

TABLE 1
IMPORTANT FACTORS AND THEIR LEVELS

S. No	Factor	Unit	Notation	Levels				
				-1.682	-1	0	+1	+1.682
1	Pressure applied	N/mm^2	A	4.0	5.0	6.0	7.0	8.0
2	Temperature	ºC	B	40.0	45.0	50.0	55.0	60.0
3	Injection hole Diameter	mm	C	6.0	7.0	8.0	9.0	10.0

TABLE 2
DESIGN MATRIX AND EXPERIMENTAL RESULTS

Ex. No	Coded values			Actual Values			Fracture length (mm)
	Pressure applied (A)	Temperature (B)	Injection hole diameter (C)	Pressure applied (A)	Temperature (B)	Injection hole diameter (C)	
1	-1	-1	-1	5.00	45.00	7.00	210
2	+1	-1	-1	7.00	45.00	7.00	250
3	-1	+1	-1	5.00	55.00	7.00	200
4	+1	+1	-1	7.00	55.00	7.00	400
5	-1	-1	+1	5.00	45.00	9.00	240
6	+1	-1	+1	7.00	45.00	9.00	350
7	-1	+1	+1	5.00	55.00	9.00	360
8	+1	+1	+1	7.00	55.00	9.00	580
9	-1.682	0	0	4.32	50.00	8.00	220
10	+1.682	0	0	7.68	50.00	8.00	460
11	0	-1.682	0	6.00	41.59	8.00	210
12	0	+1.682	0	6.00	58.41	8.00	410
13	0	0	-1.682	6.00	50.00	6.32	260
14	0	0	+1.682	6.00	50.00	9.68	420
15	0	0	0	6.00	50.00	8.00	390
16	0	0	0	6.00	50.00	8.00	420
17	0	0	0	6.00	50.00	8.00	420
18	0	0	0	6.00	50.00	8.00	350
19	0	0	0	6.00	50.00	8.00	430
20	0	0	0	6.00	50.00	8.00	330

Developing an empirical relationship

In the present investigation, to correlate experimental test parameters and the Fracture length in Hydrofraturing process, a second order quadratic model was developed. The response (Fracture length) is a function of pressure applied in N/mm^2 (A), Temperature in ºC (B) and Injection hole diameter in mm (C) and it could be expressed as,

$$\text{Fracture length (FL)} = f\{A,B,C) \quad (2)$$

The empirical relationship must include the main and interaction effects of all factors and hence the selected polynomial is expressed as follows:

$$Y = b0 + \sum bi\ xi + \sum bii\ xi^2 + \sum bij\ xi\ xj \quad (3)$$

For three factors, the selected polynomial could be expressed as

Fracture length (FL) = $b_0 + b_1(A) + b_2(B) + b_3(C)+b_{11}(A^2)+b_{22}(B^2)+b_{33}(C^3)+b_{12}(AB)+b_{13}(AC)+b_{23}(BC)$ (4) (B.Guruprasad 2013)

Where b_0 is the average of responses (Fracture length) and b_1, b_2, b_3, . . b_{11}, b_{12}, b_{13}, . . . b_{22}, b_{23}, b_{33}, are the coefficients that depend on their respective main and interaction factors, which were calculated using the expression given below

$$B_i = \sum(X_i,Y_i)/n \quad (5)$$

Where ‘i’ varies from 1 to n, in which Xi is the corresponding coded value of a factor and Yi is the corresponding response output value (Fracture length) obtained from the experiment and ‘n’ is the total number of combination considered. All the coefficients were obtained applying central composite face centered design using the Design Expert statistical software package (Trial version 8.0.1). The significance of each coefficient was determined by Student’s t test and p values, which are listed in Table 3.

TABLE 3
ESTIMATED REGRESSION COEFFICIENTS

Factor	Estimated co efficient
Intercept	3.90
A-Pressure	0.71
B-Temperature	0.61
C-Injection hole diameter	0.54
AB	0.34
AC	0.11
BC	0.26
A^2	-0.18
B^2	-0.29
C^2	-0.18

The values of “Prob>F” less than 0.0500 indicate that model terms are significant. In this case, A, B, C, AB, BC, A^2, B^2 and C^2 are significant model terms. The values greater than 0.10 indicates that model terms are not significant. The results of multiple linear regression coefficients for the second-order response surface model are given in Table 4. The final empirical relationship was constructed using only these coefficients, and the developed final empirical relationship is given below:

$$\text{Frature Length} = \{+3.90+0.71*A +0.61*B+0.54*C+0.34*A*B+0.26*B*C -0.18*A^2-0.29*B^2 -0.18*C^2\}\ \text{mm} \quad (6)$$ (B.Guruprasad 2013)

TABLE 4
ANOVA TEST RESULTS

Source	Sum of squares	df	Mean square	F Value	p-value prob.>F	
Model	19.33	9	2.15	22.95	< 0.0001	significant
A-Pressure	6.94	1	6.94	74.16	< 0.0001	
B-Temperature	5.00	1	5.00	53.43	< 0.0001	
C-Injection hole diameter	4.00	1	4.00	42.74	< 0.0001	
AB	0.91	1	0.91	9.74	0.0109	
AC	0.10	1	0.10	1.08	0.3228	
BC	0.55	1	0.55	5.89	0.0356	
A^2	0.48	1	0.48	5.12	0.0472	
B^2	1.20	1	1.20	12.80	0.0050	
C^2	0.48	1	0.48	5.12	0.0472	
Residual	0.94	10	0.094			
Lack of Fit	0.076	5	0.015	0.088	0.9907	not significant
Pure Error	0.86	5	0.17			
Cor.Total	20.27	19				

Std. Dev.	0.31	R-Squared	0.9538
Mean	3.46	Adj R-Squared	0.9123
C.V. %	8.85	Pred R-Squared	0.9081
PRESS	1.86	Adeq Precision	17.344

df -degrees of freedom, CV- coefficient of variation, F- Fisher's ratio, p- probability

The Analysis of Variance (ANOVA) technique was used to find the significant main and interaction factors. The results of second order response surface model fitting in the form of Analysis of Variance (ANOVA) are given in Table 3.The determination coefficient (r^2) indicated the goodness of fit for the model. The Model F-value of 22.95 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. The values of "Prob > F" less than 0.0500 demonstrates a very high significance for the regression model. In this case A, B, C, AB, BC, A^2, B^2, C^2 are significant model terms. The values greater than 0.1000 indicate the model terms are not significant. If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model. The goodness of fit of the model was checked by the determination coefficient (R^2). The coefficient of determination (R^2) was calculated to be 0.9538 for response. This implies that 95.38% of experimental data confirms the compatibility with the data predicted by the model, and the model does not explain only 4.62% of the total variations. The R^2 value is always between 0 and 1, and its value indicates aptness of the model. For a good statistical model, R^2 value should be close to 1.0. The adjusted R^2 value reconstructs the expression with the significant terms. The value of the adjusted determination coefficient (Adj R^2=0.9123) is also high to advocate for a high significance of the model. The Pred. R^2 is 0.9081 that implies that the model could explain 90% of the variability in predicting new observations. This is in reasonable agreement with the Adj R^2 of 0.9123. The value of coefficient of variation is also low as 8.85% indicate that the deviations between experimental and predicted values are low. Adeq precision measures the signal to noise ratio. A ratio greater than 4 is desirable. In this investigation, the ratio is 17.344, which indicates an adequate signal. This model can be used to navigate the design space. The normal probability of the Fracture length shown in Fig. 2 reveals the residuals were falling on the straight line, which meant that the errors were distributed normally. All of this indicated an excellent suitability of the regression model. Each of the observed values compared with the experimental values shown in Fig. 3.

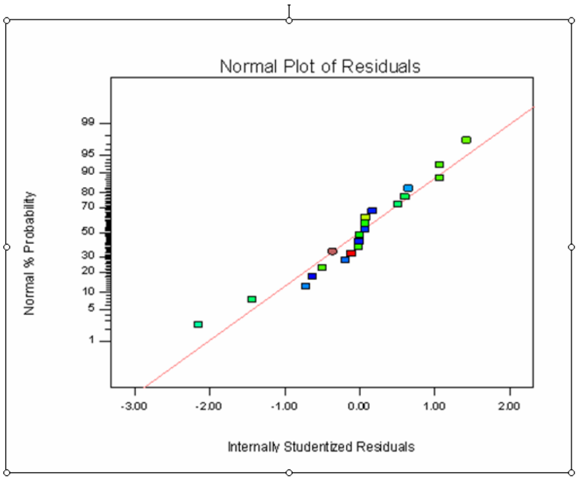

Fig. 2. Normal probability plot.

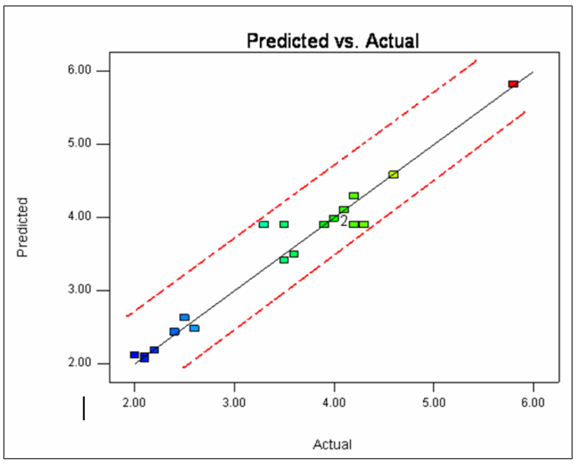

Fig. 3. Correlation graph for response (Fracture length)

Optimizing the Hydrofraturing process parameters

The response surface methodology (RSM) was used to optimize the parameters in this study. RSM is a collection of mathematical and statistical techniques that are useful for designing a set of experiments, developing a mathematical model, analyzing for the optimum combination of input parameters, and expressing the values graphically (Khuri AI 1996). To obtain the influencing nature and optimized condition of the process on Hydrofraturing, the surface plots and contour plots which are the indications of possible independence of factors have been developed for the proposed empirical relation by considering two parameters in the middle level and two parameters in the x- and y-axes as shown in Fig.5. These response contours can help in the prediction of the response for any zone of the experimental domain (Tien CL 2006). The apex of the response plot shows the maximum achievable Fracture length (mm).

A contour plot is produced to display the region of the optimal factor settings visually. For second-order responses, such a plot can be more complex compared to the simple series of parallel lines that can occur with first-order models. Once the stationary point is found, it is usually necessary to characterize the response surface in the immediate vicinity of the point. Characterization involves identifying whether the stationary point is a minimum response or maximum response or a saddle point. To classify this, it is most straightforward to examine it through a contour plot. Contour plots play a very important role in the study of a response surface. It is clear from Fig.5 that the Fracture length increases with the increase of applied pressure (N/mm^2), Temperature (ºC) and Injection hole diameter (mm).

By analyzing the response surfaces and contour plots in Fig 5, the maximum achievable fracture length (mm) value is found to be 580mm. The corresponding parameters that yielded this maximum value are Temperature 55ºC and Injection hole diameter 9mm. Contributions made by the process parameters on fracture length (mm) can be ranked (Phillip JR 1988) from their respective F ratio value which was seen in Table 3, provided the degrees of freedom are same for all the input parameters. The higher F ratio value implies that the respective term is more significant and vice versa. From the F ratio values, it can be concluded that pressure (N/mm^2) is contributing more on fracture length (mm), and it is followed by temperature (ºC) and injection hole diameter (mm) for the range considered in this investigation. . A maximum Fracture length (mm) of 580 mm obtained under the maximum value of applied pressure 7 N/mm^2 , Temperature 55ºC and Injection hole diameter 9 mm during the experimental work shown in Fig 4.

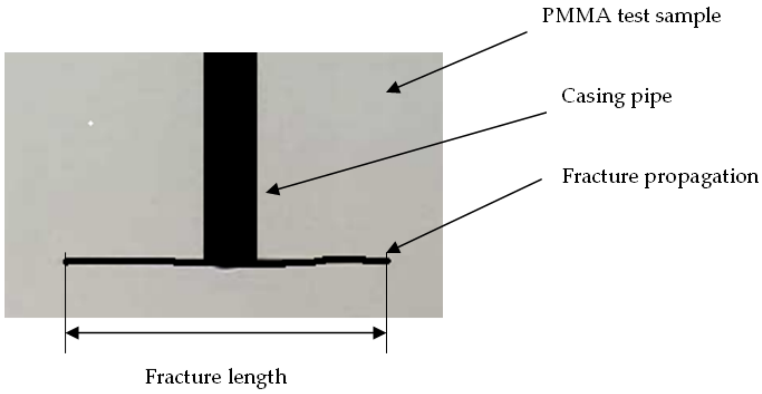

Fig. 4 Propagation of Fracture

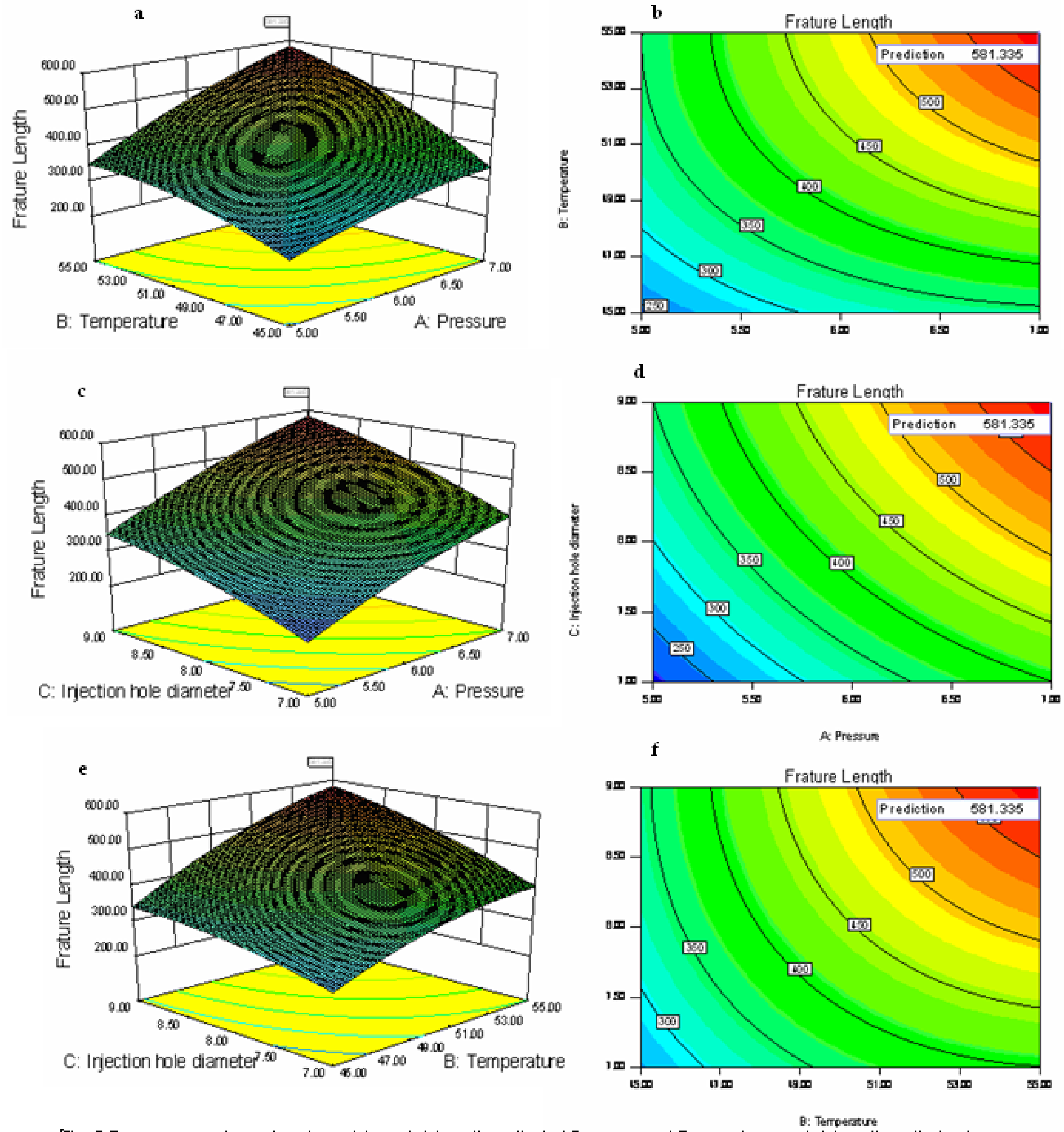

Fig. 5 Response graphs and contour plots. a, b Interaction effect of Pressure and Temperature. c, d Interaction effect of Pressure and Injection hole diameter e, f Interaction effect of Temperature and Injection hole.

Sensitivity Analysis

Sensitivity analysis is an important tool to quantify the influence of input process parameters on the output response. This type of analysis can also be used to control the input parameters during hydrofraturing process as if they are more sensitive on output response. Mathematically, sensitivity of an objective function with respect to a design variable is the partial derivative of that function with respect to its variables. The sensitivity Equations 7,8,9 represent the sensitivity on fracture length in mm for Pressure applied, Temperature and injection hole diameter respectively.

$$\partial(FL)/\partial A = [(0.71)+(0.34*B)-[2*(0.18*A)] \quad (7)$$

$$\partial(FL)/\partial B = [(0.61)+(0.34*A)+(0.26*C)-(2*(0.29*B))] \quad (8)$$

$$\partial(FL)/\partial C = [(0.54)+(0.26*B)-[2*(0.18*C)] \quad (9)$$

Sensitivity is analyzed here using the partial derivatives of Equations 7 through 9. Namely, positive sensitivity values imply an increment in the objective function by a small change in design parameter, whereas negative values state the opposite (Karaoglu .S 2007). To evaluate sensitivities, each input parameter should be varied while keeping all other input parameters constant to see how the output parameters react to these variations. An output parameter would be considered very sensitive with respect to a certain input parameter if a large change of the output parameter value is observed. Sensitivities of process parameters on fracture length are presented in Table 5.

Table 5
Sensitivity analysis of fracture length

Ex. No	Actual Values			Fracture length (mm)	Sensitivity		
	Pressure applied (A)	Temperature (B)	Injection hole diameter (C)		∂(FL)/∂A	∂(FL)/∂B	∂(FL)/∂C
1	5.00	45.00	7.00	210	16.21	-24.95	10.66
2	7.00	45.00	7.00	250	15.91	-24.61	10.66
3	5.00	55.00	7.00	200	15.55	-24.27	10.66
4	7.00	55.00	7.00	400	15.19	-23.93	10.66
5	5.00	45.00	9.00	240	14.83	-23.59	10.66
6	7.00	45.00	9.00	350	12.15	-18.47	8.06
7	5.00	55.00	9.00	360	13.85	-21.37	9.36
8	7.00	55.00	9.00	580	15.55	-24.37	10.66
9	4.32	50.00	8.00	220	17.25	-27.17	11.96
10	7.68	50.00	8.00	460	18.95	-30.07	13.26
11	6.00	41.59	8.00	210	15.55	-24.79	11.38
12	6.00	58.41	8.00	410	15.55	-24.53	11.02
13	6.00	50.00	6.32	260	15.55	-24.27	10.66
14	6.00	50.00	9.68	420	15.55	-24.01	10.30
15	6.00	50.00	8.00	390	15.55	-23.75	9.94
16	6.00	50.00	8.00	420	15.55	-24.27	10.66
17	6.00	50.00	8.00	420	15.55	-24.27	10.66
18	6.00	50.00	8.00	350	15.55	-24.27	10.66
19	6.00	50.00	8.00	430	15.55	-24.27	10.66
20	6.00	50.00	8.00	330	15.55	-24.27	10.66

Figure 6 (a–c) shows the sensitivity of fracture length for the Pressure applied, Temperature and injection hole diameter respectively on fracture length. From Fig. 6(a) it can be seen that the variation of applied pressure causes large changes of Fracture length (mm) and also higher than that of other parameters. Considering the changes of Fracture length, the sensitivity of Hydrofraturing process parameters can be ranked as follows: the applied pressure during the process is more sensitive followed by Injection hole diameter and Temperature respectively.

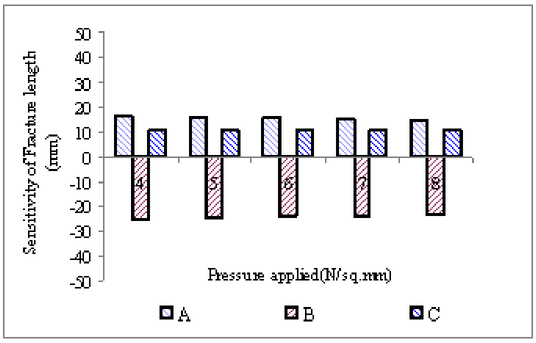

Fig.6.(a)

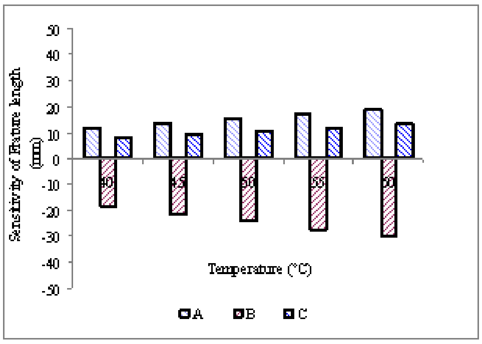

Fig.6.(b)

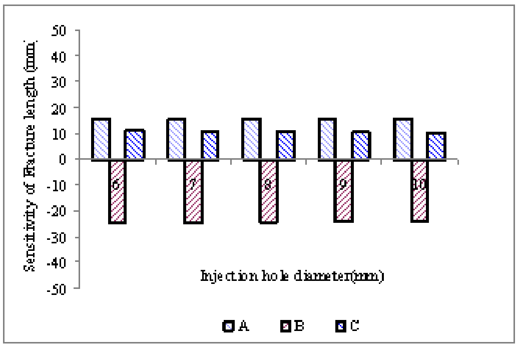

Fig.6.(c)

Fig.6 Sensitivity graphs. (a) Sensitivity of Fracture length for Pressure applied (b) Sensitivity of Fracture length for Temperature, (c) Sensitivity of Fracture length for Injection hole diameter.

Conclusion

As the Fracture length is controlled by the selection of process parameters, Fracture length has been evaluated under different processing conditions using three factors and a central composite rotatable design matrix. The sensitivity information could be useful to control the process parameters during hydrofraturing and the important results were concluded as below

1. A nonlinear empirical relationship was developed to predict the Fracture length (mm) as a mechanical property in hydrofraturing process incorporating parameters at 95% confidence level.

2. A maximum Fracture length (mm) of 581.335 mm could be attained under the maximum value of Pressure of 7N/mm^2, Temperature of 55ºC and Injection hole diameter of 9 mm.

3. Of the three process parameters investigated, the applied pressure (N/mm^2) found to have greater influence on Fracture length (mm) followed by Temperature (ºC) and Injection hole diameter (mm).

4. From the sensitivity analysis, it is found that the applied pressure (N/mm^2) is the most sensitive process parameter followed by Injection hole diameter (mm) and Temperature (ºC).

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